Brief information about the project

Name of the project	AP14869876 «Axially symmetric gravitational configurations» (0122PK00619)
Relevance	The study of astrophysical compact objects is one of the main goals of relativistic astrophysics, which can be considered as a direct application of Einstein's theory of general relativity. In general, the concept of astrophysical compact objects includes all the objects that are small for their mass such as planet-like objects, stars, white dwarfs, neutron stars, other exotic dense stars, and black holes. In this project, we will focus on the study of white dwarfs for which it is expected that relativistic effects play an important role. In particular, we are interested in finding out how the quadrupole moment, which is a measure of the deviation of the object from spherical symmetry, affects the physical properties of compact objects. Moreover, we plan to analyze this problem by considering the entire spacetime, i.e., the interior and exterior spacetimes that are joined by using the C^3 matching procedure. The physical properties of white dwarfs, in turn, can be obtained from the integration of the conservation laws, which are interpreted as the conditions for hydrostatic equilibrium of the star. To our knowledge, the consideration of the quadrupole moment, as an additional parameter that explicitly enters the equilibrium equations and the geometry of spacetime, is new. In fact, most of the studies of equilibrium conditions in relativistic astrophysics are limited to the case of spherically symmetric sources. ***The novelty of the present project*** is the way we consider the quadrupole and the axial symmetry in the metrics that determine the geometric structure of spacetime. The study of relativistic compact objects and their physical properties is very important in order to better understand our Universe. Since the formulation of general relativity as a theory of the gravitational field, thousands of research projects have been dedicated to the study of compact objects. This is a proof of the importance of the present project in the context of astrophysics and science, in general.
Purpose	The goal of the project is to study biosurfactant-producing microorganisms and their applicability for enhanced oil recovery.
Objectives	***Task 1:*** investigation of *axisymmetric* gravitational field of compact objects with the quadrupole and deformation of the surface of the object; ***Task 2:*** Investigation of the equilibrium conditions of the interior structure of the compact object with quadrupole. **2.1** Integrate the equilibrium conditions: a) with the Chandrasekhar equation of state; b) with the Salpeter equation of state; **2.2** Integrate the equilibrium conditions with the Feynman-Metropolis-Teller equation of state; find the effective baratropic equation of state for the white dwarfs;

	Tasks 3: Applications and analyze the physical significance of the results: **3.1** Determination of the mass-radius and mass-density relations for each equation of state from previous studies. **3.2** Comparison with the observational data to determine the value of the quadrupole moment for several white dwarfs. *The 1st task includes*: we will use the static quadrupolar metric (q-metric). This metric has been interpreted as the simplest generalization of the Schwarzschild metric, which is axially symmetric and includes an additional parameter *q* that determines the quadrupole moment of the source. To take into account the *rotation* of the source, we will consider a stationary generalization of the q-metric for the exterior gravitational field of rotating compact objects. The explicit form of these metric functions can be obtained from the Ernst potential derived explicitly in a previous work [1]. The calculations can be carried out either in spherical coordinates or in cylindrical coordinates. We will analyze the interior and exterior spacetimes using the C^3 match ing procedure. *The 2nd task* is to describe the interior gravitational field, we will use a particular line element proposed by us in [2] and investigated in [3]. One of the advantagesof this line element is that the corresponding conservation law equations take a very simple form. This conservation laws represents the hydrostatic relativistic equilibrium conditions, and we will integrate the hydrostatic relativistic equilibrium conditions with the equation of states to construct the models of compact objects. In additionally, we will test the solutions for the all-energy conditions, i.e., strong, weak and null energy conditions; *The 3rd task* is the astrophysical applications of the 1st and the 2nd tasks. For each task, we shall seek analyze the physical significance of the results, which will be useful for the construction of particular models of compact objects and to determine the value of the quadrupole moment for several white dwarfs, determine the mass-radius and mass-density relations planned for the 3rd year of the project implementation.
Expected and achieved results	***Task 1:*** investigation of *axisymmetric* gravitational field of compact objects with the quadrupole and deformation of the surface of the object; For Task 1: The axisymmetric gravitational field of compact objects with a quadrupole and deformation of the object's surface was studied. Axisymmetric solutions of Einstein's equations were obtained, taking into account the main characteristics of compact objects, such as rotation about a fixed axis and deformation of the object's surface. When considering a linear approximation of the deformations of compact objects characterized by a quadrupole moment, the effective refractive index was obtained. An explicit form of metric functions for the stationary case was obtained from

	consideration of the Ernst potential. Internal and external spacetime were analyzed using the C3 matching procedure. ***Task 2:*** Investigation of the equilibrium conditions of the interior structure of the compact object with quadrupole. **1.1** Integrate the equilibrium conditions: a) with the Chandrasekhar equation of state; b) with the Salpeter equation of state; **1.2** Integrate the equilibrium conditions with the Feynman-Metropolis-Teller equation of state; find the effective baratropic equation of state for the white dwarfs; The solution to the internal field equations for a compact object has been found: According to Task 2.1: The influence of the quadrupole parameter q- on the Chandrasekhar and Salpeter equation of state is studied. According to Task 2.2: The influence of the quadrupole parameter q on the Feynman-Metropolis-Teller equation of state was studied. ***Tasks 3:*** Applications and analyze the physical significance of the results (2024) **3.1** Determination of the mass-radius and mass-density relations for each equation of state from previous studies. **3.2** Comparison with the observational data to determine the value of the quadrupole moment for several white dwarfs.
Research team members with their identifiers (Scopus Author ID, Researcher ID, ORCID, if available) and links to relevant profiles	1. **Beissen Nurzada**, Education: Al-Farabi Kazakh National University Almaty, RK. Degree: Candidate's degree in Physical and Mathematical sciences (2005). Work experience in the direction of the project more than 20 years (physics and astronomy); ScopusID: https://www.scopus.com/authid/detail.uri?authorId=26530753300 (Scopus h-index=3). WoS/Publons ID: https://publons.com/researcher/2754418/nurzada-beissen/ (WoS/Publons h-index=3). ORCID: http://orcid.org/0000-0002-1957-2768; Position and role in the project: General management of the Project, coordination of the Project, development and testing of computer code, preparation of scientific articles, writing reports on the Project. 2. **Abishev Medeu**, Education: Al-Farabi Kazakh National University Almaty, RK. Degree: D.Sc. (2010). Work experience in the direction of the project more than 20 years (physics and astronomy); ScopusID: https://www.scopus.com/authid/detail.uri?authorId=26530759900 (Scopus h-index=3). WoS/Publons ID: https://publons.com/researcher/1906391/medeu-abishev/ (WoS/Publons h-index=3). ORCID: https://orcid.org/0000-0003-3602-6934; Position and role in the project: development and testing of computer code, preparation of scientific articles, writing reports on the project. 3. **Toktarbay Saken**, Education: Al-Farabi KazNU (Bachelor, 2009, Kazakhstan); Tomsk Polytechnic National Research University (Master, 2011, Russia); Al-Farabi KazNU (PhD, 2016,

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List of publications with links to them	*Scopus и Web of Science.* 1. Beissen N, Abishev M, Khassanov M, Aitassov T, Mamatova S, Toktarbay S. Stability Properties of Geometrothermodynamic Cosmological Models. *Entropy*. 2023; 25(10):1391. https://doi.org/10.3390/e25101391 **ISSN**:1099-4300, **, 89%, Q2** 2. Beissen N, Utepova D, Abishev M, Quevedo H, Khassanov M, Toktarbay S. Gravitational Refraction of Compact Objects with Quadrupoles. Symmetry. 2023; 15(3):614. https://doi.org/10.3390/sym15030614 **ISSN:2073-8994, 93%, Q2** *In a journal included in KOKSON MES RK* NA Beissen, D Utepova, A Muratkhan, A Orazymbet, M Khassanov, S Toktarbay. Application of GBT theorem for Gravitational deflection of light by Compact Objects // Recent Contributions to Physics, Vol. 84 No. 1, 2023. *International conference* 1. International Conference ABDILDIN READINGS (ACTUAL PROBLEMS OF MODERN PHYSICS), Al Farabi Kazakh National University, April 12–15, 2023, Almaty

	2. Conference in Gyeongju (South Korea) “15th International Conference on Gravitation, Astrophysics and Cosmology (ICGAC15)” from July 3 to July 7, 2023
Patents	-